

Unusual light spectra from a two-level atom in squeezed vacuum

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Abstract

We investigate the interaction of an atom with a multi-channel squeezed vacuum. It turns out that the light coming out in a particular channel can have anomalous spectral properties, among them asymmetry of the spectrum, absence of the central peak as well as central hole burning for particular parameters. As an example plane-wave squeezing is considered. In this case the above phenomena can occur for the light spectra in certain directions. In the total spectrum these phenomena are washed out.

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1 Introduction

The interaction of squeezed light with atoms has found considerable theoretical attention in the last years. As pointed out in the seminal paper of Gardiner [1], squeezed light, either in the white noise limit or broadband colored noise [2], can drastically alter the radiative properties of atoms and can, in principle, reduce substantially the spectral linewidth of emitted light. Gardiner's paper has stimulated a large amount of work on the interaction of squeezed light with atoms. We refer to the detailed review of Parkins [3] for references and an account of work up to 1993. For more recent work see, e.g., Refs. [4, 5] and references therein.

Gardiner [1, 6] considered a one-dimensional like situation in which only a single channel is squeezed and where the atom couples only to this channel. Here a channel means a set of modes ranging over all frequencies with the remaining quantum numbers characterizing the modes kept fixed [1]. For example, a channel may consist of all modes with fixed angular momentum and parity for a multipole expansion of the field, or of all plane wave modes with a given direction and polarization.

An investigation of resonance fluorescence spectra of atoms interacting with a (single-channel) squeezed vacuum plus a laser was initiated by Carmichael, Lane, and Walls [7] (for further references see the review [3] and Refs. [4, 5]). Smart and Swain [8, 9] have pointed out the existence of interesting structures in these spectra. In Ref. [11, 10] multi-channel squeezing and associated correlation functions for three-level atoms were studied.

In this paper we consider the spectral effects of a multi-channel squeezed vacuum in the white noise limit on a two-level atom. For the atomic correlation functions and the *total* spectrum of all outgoing light a multi-channel squeezed vacuum leads to analogous results as a single-channel squeezed vacuum with appropriate parameters. However, in a multi-channel situation one can observe not only the total spectrum but also the light spectrum in individual channels. It turns out that — due to interference of the (quantized) light scattered from the atom with the squeezed vacuum — these spectra can show unexpected features which are not visible in the total spectrum, e.g., a possible asymmetry, absence of the central peak as well as central hole burning for particular parameters. By the same arguments as in Ref. [2] we expect these features to persist also for only approximate white-noise squeezing.

In Section 2 below the general case of multi-channel squeezed white noise interacting with a two-level atom is treated. The spectrum is calculated in terms of a background, scattered, and interference part.

In Section 3 we treat in detail an example in which the channels consist of plane waves with fixed directions and polarizations. In this case one has a divergence of the background term when calculated in terms of photon numbers, and we therefore use the spectral Poynting vector to calculate the spectrum for given position and direction of observation of the spectral analyzer. In this case the above phenomena like asymmetry, central hole burning etc. can occur for the light spectra in certain directions.

In Section 4 we discuss our results in detail, in particular the question of interference, and point out a possible connection, probably more formal than directly physical, with the results of Ref. [9].

2 Spectra for general multi-channel squeezing

We consider a two-level atom coupled to the electromagnetic field in three-dimensional space. The free Hamiltonians of the atom and the field are given by

$$H_A = \hbar\omega_0 \sigma^+ \sigma^- \quad H_F = \int_0^\infty d\omega \sum_\alpha \hbar\omega a_\alpha^\dagger(\omega) a_\alpha(\omega) , \quad (1)$$

where ω_0 denotes the atomic transition frequency, $\sigma^+ = |+\rangle\langle-|$ and $\sigma^- = |- \rangle\langle+|$ are the atomic raising and lowering operators and the $a_\alpha(\omega)$ are the annihilation

operators of the field obeying the commutation relations

$$[a_{\alpha_1}(\omega_1), a_{\alpha_2}^\dagger(\omega_2)] = \delta_{\alpha_1\alpha_2} \delta(\omega_1 - \omega_2) . \quad (2)$$

The index α stands for all quantum numbers of the chosen modes apart from their frequency and thus characterizes a channel. These are, for example, parity and angular momentum quantum numbers if the multipole expansion of the field is used, or the direction of propagation together with the polarization for plane wave modes. In the rotating-wave approximation, the interaction Hamiltonian may be written as

$$H_{\text{AF}} = \frac{i\hbar}{\sqrt{2\pi}} \int_0^\infty d\omega \sum_\alpha g_\alpha(\omega) a_\alpha^\dagger(\omega) \sigma^- + \text{h.c.} \quad (3)$$

with possibly complex coupling coefficients $g_\alpha(\omega)$, which we decompose as

$$g_\alpha(\omega) \equiv \sqrt{\gamma_\alpha(\omega)} e^{i\phi_\alpha(\omega)} . \quad (4)$$

Going over to the interaction picture leads to the Hamiltonian

$$H_I(t) = i\hbar \sum_\alpha \sqrt{\gamma_\alpha} [b_\alpha^\dagger(t) \sigma^- - b_\alpha(t) \sigma^+] \quad (5)$$

with $\gamma_\alpha \equiv \gamma_\alpha(\omega_0)$ and

$$b_\alpha(t) \equiv \frac{1}{\sqrt{2\pi}} \int_0^\infty d\omega \left(\frac{\gamma_\alpha(\omega)}{\gamma_\alpha(\omega_0)} \right)^{1/2} e^{-i\phi_\alpha(\omega)} e^{-i(\omega-\omega_0)t} a_\alpha(\omega) . \quad (6)$$

Assuming that the requirements for applying the Markov approximation are satisfied we replace the second factor in the commutator

$$[b_\alpha(s), b_\beta^\dagger(t)] = \delta_{\alpha\beta} \cdot \frac{1}{2\pi} \int_0^\infty d\omega \frac{\gamma_\alpha(\omega)}{\gamma_\alpha(\omega_0)} e^{-i(\omega-\omega_0)(s-t)} \approx \delta_{\alpha\beta} \cdot \delta(s-t) \quad (7)$$

by a δ -function (cf. [12, 6]).

The radiation field is supposed to be initially in a pure broadband squeezed vacuum state with the atomic frequency ω_0 as central frequency. In general, for such a state the second order moments of the $b_\alpha(\omega)$ read in the white noise limit [12, 6]

$$\langle b_\alpha(s) b_\beta(t) \rangle = \mathcal{M}_{\alpha\beta} \delta(s-t) \quad \langle b_\alpha^\dagger(s) b_\beta(t) \rangle = \mathcal{N}_{\alpha\beta} \delta(s-t) \quad (8)$$

with

$$\mathcal{M}^T = \mathcal{M} \quad \mathcal{N}^\dagger = \mathcal{N} \quad \mathcal{M}^\dagger \mathcal{M} = \mathcal{N}(\mathcal{N} + 1) . \quad (9)$$

If one omits factors of the form $[\gamma_\alpha(\omega)/\gamma_\alpha(\omega_0)]^{1/2}$ this leads to

$$\begin{aligned} \langle a_\alpha(\omega_1) a_\beta(\omega_2) \rangle &= \mathcal{M}_{\alpha\beta} \delta(2\omega_0 - \omega_1 - \omega_2) e^{i[\phi_\alpha(\omega_1) + \phi_\beta(\omega_2)]} \\ \langle a_\alpha^\dagger(\omega_1) a_\beta(\omega_2) \rangle &= \mathcal{N}_{\alpha\beta} \delta(\omega_1 - \omega_2) e^{i[-\phi_\alpha(\omega_1) + \phi_\beta(\omega_2)]} \end{aligned} \quad (10)$$

for the moments of the $a_\alpha(\omega)$. In the following Eq. (8) will be used as definition of squeezed white noise.

We shall assume in the following that \mathcal{M} and \mathcal{N} are diagonal for the given modes,

$$\mathcal{M}_{\alpha\beta} = \delta_{\alpha\beta} M_\alpha \quad \mathcal{N}_{\alpha\beta} = \delta_{\alpha\beta} N_\alpha . \quad (11)$$

One can imagine a state of this kind as being produced by independently squeezing modes with different α , e.g., by coupling them to different parametric oscillators [13]. For a pure state, as considered here, the assumption (11) seems not to be very restrictive. See [14, 15], where for a finite number of modes the question is discussed to what extent second order moments can be simplified by choosing appropriate modes.

The fluorescence spectrum of an atom illuminated by squeezed white noise has been studied by Gardiner [1] and has since become textbook material [6]. In Gardiner's article and in a large part of the following work it was supposed that initially only a single channel of the radiation field is squeezed and that the atom couples only to this channel. In our notation, this means there is a particular α , $\alpha = 0$, say, with

$$M_\alpha = M\delta_{\alpha,0} \quad N_\alpha = N\delta_{\alpha,0} \quad \gamma_\alpha = \gamma\delta_{\alpha,0} . \quad (12)$$

Such a situation will, a little imprecisely, be called *one-dimensional* in the following.

We now want to calculate the spectrum $\mathcal{S}_\alpha(\omega)$ that would be observed by a spectral analyzer coupled to the modes (α, ω) with α fixed. (For plane wave modes this would simply be the spectrum of light with a certain polarization observed in a certain direction.) One could of course also observe and determine other spectra, e.g., the spectrum in a channel which is a superposition of different α 's. Our procedure easily carries over to this situation. To determine $\mathcal{S}_\alpha(\omega)$ we shall adapt the procedure of Gardiner [1, 6] to multi-channel squeezing.

At a finite time the spectrum is proportional to the expectation value of the photon number operator $a_\alpha^\dagger(\omega)a_\alpha(\omega)$. The stationary spectrum is given by

$$\mathcal{S}_\alpha(\omega) = \lim_{T \rightarrow \infty} \frac{1}{2\pi T} \int_0^T dt_1 \int_0^T dt_2 e^{-i(\omega-\omega_0)(t_1-t_2)} w_\alpha(t_1, t_2) , \quad (13)$$

with the two-time correlation function

$$w_\alpha(t_1, t_2) = \langle b_\alpha^\dagger(t_1)_{\text{out}} b_\alpha(t_2)_{\text{out}} \rangle , \quad (14)$$

and can be written as the Fourier transform

$$\mathcal{S}_\alpha(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\tau e^{-i(\omega-\omega_0)\tau} w_\alpha(\tau) = \frac{1}{\pi} \text{Re} \int_0^{\infty} d\tau e^{-i(\omega-\omega_0)\tau} w_\alpha(\tau) \quad (15)$$

of the stationary correlation function

$$w_\alpha(\tau) = \lim_{t_2 \rightarrow \infty} w_\alpha(\tau + t_2, t_2) . \quad (16)$$

The operators $b_\alpha(s)_{\text{out}}$ appearing here denote the limits

$$b_\alpha(s)_{\text{out}} = \lim_{t \rightarrow \infty} b_\alpha(s)_t , \quad (17)$$

where the subscript t stands for the time evolution in the interaction picture,

$$X_t = U_I(t, 0)^\dagger X U_I(t, 0) . \quad (18)$$

Since the $b_\alpha(s)_t$ obey the equations of motion

$$\frac{d}{dt} b_\alpha(s)_t = \frac{i}{\hbar} [H_I(t)_t, b_\alpha(s)_t] = \delta(t - s) \sqrt{\gamma_\alpha} \sigma_t^- \quad (19)$$

one has

$$b_\alpha(s)_t = b_\alpha(s) + \vartheta(t - s) \sqrt{\gamma_\alpha} \sigma_s^- , \quad (20)$$

where ϑ denotes the Heaviside function, and in particular

$$b_\alpha(s)_{\text{out}} = b_\alpha(s) + \sqrt{\gamma_\alpha} \sigma_s^- . \quad (21)$$

Inserting this into Eq. (13) yields a decomposition of the spectrum into three parts, corresponding to the correlation functions

$$\begin{aligned} w_\alpha^B(t_1, t_2) &\equiv \langle b_\alpha^\dagger(t_1) b_\alpha(t_2) \rangle & w_\alpha^S(t_1, t_2) &\equiv \gamma_\alpha \langle \sigma_{t_1}^+ \sigma_{t_2}^- \rangle \\ w_\alpha^I(t_1, t_2) &\equiv \sqrt{\gamma_\alpha} \langle b_\alpha^\dagger(t_1) \sigma_{t_2}^- + \sigma_{t_1}^+ b_\alpha(t_2) \rangle , \end{aligned} \quad (22)$$

which will be called the background, scattered, and interference part, respectively.

For the background part one finds immediately

$$w_\alpha^B(t_1, t_2) = w_\alpha^B(t_1 - t_2) = N_\alpha \delta_{\alpha\alpha} \delta(t_1 - t_2) \quad 2\pi \mathcal{S}^B(\omega) = N_\alpha \delta_{\alpha\alpha} , \quad (23)$$

as expected for the spectrum of white noise. Although $\delta_{\alpha\alpha} = 1$, it has explicitly been kept since it becomes divergent if the index α is not purely discrete, e.g., for plane wave modes. For such modes, a state with \mathcal{M}, \mathcal{N} being diagonal is an idealization just as a plane wave coherent state. It will be shown in the next section how this problem can be bypassed in a simple physical way.

The ‘mixed’ correlation function $w_\alpha^I(t_1, t_2)$ can be reduced to an expression containing only the atomic operators σ_t^\pm by means of the formulae

$$\langle b_\alpha^\dagger(t_1) \sigma_{t_2}^- \rangle = \vartheta(t_2 - t_1) \cdot \sqrt{\gamma_\alpha} \langle N_\alpha[\sigma_{t_1}^+, \sigma_{t_2}^-] - M_\alpha^*[\sigma_{t_1}^-, \sigma_{t_2}^-] \rangle \quad (24)$$

$$\langle \sigma_{t_1}^+ b_\alpha(t_2) \rangle = \vartheta(t_1 - t_2) \cdot \sqrt{\gamma_\alpha} \langle N_\alpha[\sigma_{t_1}^+, \sigma_{t_2}^-] - M_\alpha[\sigma_{t_1}^+, \sigma_{t_2}^+] \rangle \quad (25)$$

which can be derived in a similar way as in the one-dimensional case.

The remaining correlation functions of the σ^\pm can be evaluated in the stationary limit using the quantum regression theorem and the atomic master equation.

Since the calculations for the one-dimensional case in [6] can be carried over, except for some obvious modifications, we just give the results. The form of the master equation

$$\begin{aligned} \dot{\rho} = & \gamma(N+1)\left(\sigma^- \rho \sigma^+ - \frac{1}{2}\{\sigma^+ \sigma^-, \rho\}\right) + \gamma N\left(\sigma^+ \rho \sigma^- - \frac{1}{2}\{\sigma^- \sigma^+, \rho\}\right) \\ & - \gamma M \sigma^+ \rho \sigma^+ - \gamma M^* \sigma^- \rho \sigma^- \end{aligned} \quad (26)$$

remains unchanged, but the parameters γ , M , and N now are defined as

$$\gamma = \sum_{\alpha} \gamma_{\alpha} \quad M = \sum_{\alpha} \frac{\gamma_{\alpha}}{\gamma} M_{\alpha} \quad N = \sum_{\alpha} \frac{\gamma_{\alpha}}{\gamma} N_{\alpha} . \quad (27)$$

The possible values of M and N are restricted by the inequality

$$|M|^2 \leq N(N+1) , \quad (28)$$

which follows from the relations $|M_{\alpha}|^2 = N_{\alpha}(N_{\alpha}+1)$ [cf. Eq. (9)].

In the one-dimensional case γ , M , and N defined above agree with those appearing in Eq. (12), and for pure states — which we have been considering above — the equality sign would hold in Eq. (28). The full range of the parameters M and N can also be realized in this case if one uses mixed states.

By absorbing a phase into the atomic states if necessary, M can be chosen real and positive. With this convention one gets for $\tau \geq 0$ (the relation $w_{\alpha}^{S,I}(\tau) = w_{\alpha}^{S,I}(-\tau)^*$ yields the corresponding expressions for $\tau < 0$)

$$w_{\alpha}^S(\tau) = \frac{\gamma_{\alpha}}{2} \frac{N}{2N+1} \left\{ e^{-\gamma_+ \tau} + e^{-\gamma_- \tau} \right\} \quad (29)$$

$$w_{\alpha}^I(\tau) = -\frac{\gamma_{\alpha}}{2} \frac{1}{2N+1} \left\{ e^{-\gamma_+ \tau} (N_{\alpha} + M_{\alpha}) + e^{-\gamma_- \tau} (N_{\alpha} - M_{\alpha}) \right\} , \quad (30)$$

where

$$\gamma_{\pm} = \left(N \pm M + \frac{1}{2}\right) \gamma . \quad (31)$$

The complete spectrum is the Fourier transform of

$$\begin{aligned} w_{\alpha}(\tau) = & N_{\alpha} \delta_{\alpha\alpha} \delta(\tau) + \frac{\gamma_{\alpha}}{2} \frac{1}{2N+1} \\ & \times \left\{ e^{-\gamma_+ \tau} (N - N_{\alpha} - M_{\alpha}) + e^{-\gamma_- \tau} (N - N_{\alpha} + M_{\alpha}) \right\} , \end{aligned} \quad (32)$$

$$\begin{aligned} 2\pi \mathcal{S}_{\alpha}(\omega) = & N_{\alpha} \delta_{\alpha\alpha} + \frac{\gamma_{\alpha}}{2N+1} \sum_{\pm} \frac{\gamma_{\pm}}{(\omega - \omega_0)^2 + \gamma_{\pm}^2} (N - N_{\alpha} \mp \text{Re } M_{\alpha}) \\ & + 2M \gamma_{\alpha} \gamma^2 \frac{\omega - \omega_0}{[(\omega - \omega_0)^2 + \gamma_+^2][(\omega - \omega_0)^2 + \gamma_-^2]} \text{Im } M_{\alpha} . \end{aligned} \quad (33)$$

For the spectrum of all modes, $\mathcal{S}(\omega) = \sum_{\alpha} \mathcal{S}_{\alpha}(\omega)$, one obtains

$$2\pi \mathcal{S}(\omega) = N + \frac{M\gamma}{2N+1} \left\{ \frac{\gamma_-}{(\omega - \omega_0)^2 + \gamma_-^2} - \frac{\gamma_+}{(\omega - \omega_0)^2 + \gamma_+^2} \right\}, \quad (34)$$

an expression that coincides with the spectrum in the one-dimensional case. This shows that, as long as only this kind of spectrum is observed, all states satisfying (11) are equivalent to mixed states of the one-dimensional type (12); c.f. the remark following Eq. (28).

In contrast to $\mathcal{S}(\omega)$, $\mathcal{S}_{\alpha}(\omega)$ shows new features, namely in general the spectra are asymmetric and the sign and the relative weight of the peaks depend on M_{α} and N_{α} . The asymmetries are caused by the phase of M_{α} (this phase has a physical meaning as it is the relative phase of M_{α} and M) since the symmetric and antisymmetric parts of $\mathcal{S}_{\alpha}(\omega_0 + \omega')$ are the Fourier transforms of the real and imaginary part of $w_{\alpha}(\tau)$, respectively.

In figure 1 the spectrum (33) is plotted for $N_{\alpha} = N$ and increasing values of $\varphi = \arg M_{\alpha}$. For $\varphi = 0$ one obtains the spectrum of the one-dimensional case consisting of two Lorentzians, a positive narrow peak of width γ_- and a negative broad peak of width γ_+ . For $0 < \varphi < \pi$ the spectrum is asymmetric with respect to ω_0 , the asymmetry being maximal for $\varphi = \pi/2$. The relative weights of the symmetric contributions to the spectrum decrease and vanish for $\varphi = \pi/2$. For $\pi/2 < \varphi \leq \pi$, the weights increase again but the signs of the two Lorentzians are interchanged. The spectra for $\pi \leq \varphi \leq 2\pi$ coincide with those for $2\pi - \varphi$ mirrored at $\omega = \omega_0$.

For $N_{\alpha} = 0$ (and consequently $M_{\alpha} = 0$) the background and interference parts of the spectra vanish. As the scattered part does not depend on N_{α} and M_{α} , the spectra take the shape of the dotted line in figure 1 (the dashed line now representing $\mathcal{S} = 0$).

Spectra for $N_{\alpha} > N$ are plotted in figure 2. As compared to those with $N_{\alpha} = N$, the positive peaks are attenuated while the depth of the negative peaks increases.

3 Example: Squeezed plane waves

To be more specific, we shall now deal with plane wave modes in greater detail. These modes are specified by their wave-vector \mathbf{k} and their polarization $\lambda = 1, 2$, i.e., by $\omega = ck$ and $\alpha = (\hat{\mathbf{k}}, \lambda)$ with $\hat{\mathbf{k}} = \mathbf{k}/k$. In the dipole and rotating-wave approximation the interaction Hamiltonian is given by

$$H_{\text{AF}} = -\mathbf{d} \cdot \mathbf{E}^{(-)} \sigma^{-} - \mathbf{d}^* \cdot \mathbf{E}^{(+)} \sigma^{+}, \quad (35)$$

where $\mathbf{E}^{(+)} = \mathbf{E}^{(-)\dagger}$ denotes the positive frequency part of the electric field at the position $\mathbf{r} = 0$ of the atom,

$$\mathbf{E}^{(+)} = \frac{i}{(2\pi)^{\frac{3}{2}}} \int d^3\mathbf{k} \left(\frac{\hbar\omega}{2\epsilon_0} \right)^{1/2} \sum_{\lambda=1,2} \boldsymbol{\epsilon}_{\mathbf{k},\lambda}^* a_{\mathbf{k},\lambda} , \quad (36)$$

and $\mathbf{d} = \langle -|\mathbf{D}|+ \rangle$ is a matrix element of the atomic dipole operator \mathbf{D} (by parity, the static dipole moments in the states $|\pm\rangle$ vanish).

In order to apply the formulae of the previous section, the sums over α and the Kronecker symbols $\delta_{\alpha_1\alpha_2}$ have to be replaced according to

$$\sum_{\alpha} \longrightarrow \int_{S^2} d^2\hat{\mathbf{k}} \sum_{\lambda=1,2} \delta_{\alpha_1\alpha_2} \longrightarrow \delta_{\lambda_1\lambda_2} \delta^2(\hat{\mathbf{k}}_1, \hat{\mathbf{k}}_2) , \quad (37)$$

where $d^2\hat{\mathbf{k}}$ denotes the area element on the unit sphere and $\delta^2(\hat{\mathbf{k}}_1, \hat{\mathbf{k}}_2)$ the corresponding δ -function. By means of the identity

$$\delta^3(\mathbf{k}_1 - \mathbf{k}_2) = k_1^{-2} \delta(k_1 - k_2) \delta^2(\hat{\mathbf{k}}_1, \hat{\mathbf{k}}_2) \quad (38)$$

one can write

$$[a_{\mathbf{k}_1,\lambda_1}, a_{\mathbf{k}_2,\lambda_2}^\dagger] = \delta^3(\mathbf{k}_1 - \mathbf{k}_2) \delta_{\lambda_1\lambda_2} = \frac{c^3}{\omega_1^2} \cdot \delta^2(\hat{\mathbf{k}}_1, \hat{\mathbf{k}}_2) \delta_{\lambda_1\lambda_2} \cdot \delta(\omega_1 - \omega_2) \quad (39)$$

and by comparing with Eq. (2) one sees that operators with commutation relations analogous to those of the $a_\alpha(\omega)$ can be defined by

$$a_{\hat{\mathbf{k}},\lambda}(\omega) \equiv \omega c^{-\frac{3}{2}} a_{\mathbf{k},\lambda} . \quad (40)$$

The coupling coefficients appearing in a representation of H_{AF} in the form of Eq. (3) are therefore given by

$$g_{\hat{\mathbf{k}},\lambda}(\omega) = \frac{1}{2\pi} \left(\frac{\omega^3}{2\epsilon_0 \hbar c^3} \right)^{1/2} \boldsymbol{\epsilon}_{\hat{\mathbf{k}},\lambda} \cdot \mathbf{d} \equiv \sqrt{\gamma_{\hat{\mathbf{k}},\lambda}(\omega)} e^{i\phi_{\hat{\mathbf{k}},\lambda}} . \quad (41)$$

Note that their phases do not depend on ω .

As already noted, a squeezed state with \mathcal{M} , \mathcal{N} being diagonal for plane waves leads to a divergent expression in Eq. (23). One could try to circumvent this problem by subtracting the divergent background part, but a physically more satisfactory solution is to use an improved definition of the spectrum which avoids divergences automatically.

A suitable quantity for modeling the spectrum actually observed in an experiment is the spectrally resolved energy flux through a (small) surface \mathcal{A} centered at $\mathbf{r} = \mathbf{r}_0$,

$$\mathcal{S}_{\mathcal{A},\mathbf{r}_0}(\omega) = \int_{(\mathcal{A}, \mathbf{r}_0)} d\boldsymbol{\sigma} \cdot \langle \mathbf{S}(\mathbf{r}, \omega) \rangle , \quad (42)$$

where the operator $\mathbf{S}(\mathbf{r}, \omega)$ represents the ‘spectral Poynting vector’, i.e., the spectral energy flux density. More realistically, one could also use a direction-sensitive spectral analyzer, e.g., an analyzer admitting only directions in a certain cone in \mathbf{k} space, such that only radiation with directions from this cone is observed. It can be shown that, as physically expected, the scattered and interference parts of the spectrum are not influenced by this as long as the cone contains the line between the atom and the detector. For the background part the directional selection would simply result in a restriction of the integration over $\hat{\mathbf{k}}$ in Eq. (54) below to this cone.

For the operator $\mathbf{S}(\mathbf{r}, \omega)$ we shall use the expression (cf. [16])

$$\mathbf{S}(\mathbf{r}, \omega) = \lim_{T \rightarrow \infty} \frac{\epsilon_0 c^2}{T} \tilde{\mathbf{E}}_T^{(-)}(\mathbf{r}, \omega) \times \tilde{\mathbf{B}}_T^{(+)}(\mathbf{r}, \omega) + \text{h.c.} , \quad (43)$$

where $\tilde{\mathbf{E}}_T^{(\pm)}(\mathbf{r}, \omega)$ — and analogously $\tilde{\mathbf{B}}_T^{(\pm)}(\mathbf{r}, \omega)$ — is defined by

$$\tilde{\mathbf{E}}_T^{(\pm)}(\mathbf{r}, \omega) = \frac{1}{\sqrt{2\pi}} \int_0^T dt e^{\pm i\omega t} \mathbf{E}^{(\pm)}(\mathbf{r}, t) , \quad (44)$$

$\mathbf{E}^{(\pm)}(\mathbf{r}, t)$ and $\mathbf{B}^{(\pm)}(\mathbf{r}, t)$ being the Heisenberg operators of the positive and negative frequency parts of the (transversal) electric and magnetic fields,

$$\mathbf{E}^{(+)}(\mathbf{r}, t) = \frac{i}{(2\pi)^{\frac{3}{2}}} \int d^3\mathbf{k} \left(\frac{\hbar\omega}{2\epsilon_0} \right)^{1/2} \sum_{\lambda=1,2} \boldsymbol{\epsilon}_{\mathbf{k},\lambda}^* e^{i\mathbf{k}\cdot\mathbf{r}} a_{\mathbf{k},\lambda}(t) \quad (45)$$

$$\mathbf{B}^{(+)}(\mathbf{r}, t) = \frac{i}{c(2\pi)^{\frac{3}{2}}} \int d^3\mathbf{k} \left(\frac{\hbar\omega}{2\epsilon_0} \right)^{1/2} \sum_{\lambda=1,2} \hat{\mathbf{k}} \times \boldsymbol{\epsilon}_{\mathbf{k},\lambda}^* e^{i\mathbf{k}\cdot\mathbf{r}} a_{\mathbf{k},\lambda}(t) . \quad (46)$$

Introducing the correlation function [17]

$$w_{\mathcal{A},\mathbf{r}_0}(t_1, t_2) = \epsilon_0 c^2 \int_{(\mathcal{A}, \mathbf{r}_0)} d\boldsymbol{\sigma} \cdot e^{-i\omega_0(t_1-t_2)} \langle \mathbf{E}^{(-)}(\mathbf{r}, t_1) \times \mathbf{B}^{(+)}(\mathbf{r}, t_2) - \mathbf{B}^{(-)}(\mathbf{r}, t_1) \times \mathbf{E}^{(+)}(\mathbf{r}, t_2) \rangle \quad (47)$$

and its stationary limit $w_{\mathcal{A},\mathbf{r}_0}(\tau) = \lim_{t_2 \rightarrow \infty} w_{\mathcal{A},\mathbf{r}_0}(\tau + t_2, t_2)$, $\mathcal{S}_{\mathcal{A},\mathbf{r}_0}(\omega)$ can be written as a Fourier transform in the same way as $\mathcal{S}_\alpha(\omega)$ in Eqs. (13) and (15),

$$\begin{aligned} \mathcal{S}_{\mathcal{A},\mathbf{r}_0}(\omega) &= \lim_{T \rightarrow \infty} \frac{1}{2\pi T} \int_0^T dt_1 \int_0^T dt_2 e^{-i(\omega-\omega_0)(t_1-t_2)} w_{\mathcal{A},\mathbf{r}_0}(t_1, t_2) \\ &= \frac{1}{\pi} \text{Re} \int_0^\infty d\tau e^{-i(\omega-\omega_0)\tau} w_{\mathcal{A},\mathbf{r}_0}(\tau) . \end{aligned} \quad (48)$$

Since the time evolution operator $U(t) = \exp(-i\hbar^{-1}(H_A + H_F)t) U_I(t, 0)$ transforms $b_\alpha(s)$ into

$$U(t)^\dagger b_\alpha(s) U(t) = e^{-i\omega_0 t} b_\alpha(s+t)_t , \quad (49)$$

it follows from Eqs. (6) and (41) that the $a_{\mathbf{k},\lambda}(t)$ are related to the $b_{\hat{\mathbf{k}},\lambda}(t+s)_t$ by

$$e^{i\phi_{\hat{\mathbf{k}},\lambda}} e^{-i\omega_0(t+s)} b_{\hat{\mathbf{k}},\lambda}(t+s)_t = \frac{1}{\sqrt{2\pi}} \int_0^\infty d\omega \left(\frac{\omega}{\omega_0} \right)^p \omega c^{-\frac{3}{2}} e^{-i\omega s} a_{\mathbf{k},\lambda}(t) \quad (50)$$

with $p = 3/2$. Within the scope of the Markov approximation, this equation remains valid also for $p \neq 3/2$ since the factor ω/ω_0 can be replaced by unity just as $\gamma_\alpha(\omega)/\gamma_\alpha(\omega_0)$ in Eq. (7) (cf. [6], Ch. 8.1).

Because of the linearity of the above relation, the decomposition of the $b_{\hat{\mathbf{k}},\lambda}(s)_t$ according to Eq. (20),

$$\begin{aligned} b_{\hat{\mathbf{k}},\lambda}(t+s)_t &= b_{\hat{\mathbf{k}},\lambda}(t+s) + \vartheta(-s) \sqrt{\gamma_{\hat{\mathbf{k}},\lambda}} \sigma_t^- + s \\ &\equiv b_{\hat{\mathbf{k}},\lambda}(t+s)_t^{(1)} + b_{\hat{\mathbf{k}},\lambda}(t+s)_t^{(2)} , \end{aligned} \quad (51)$$

leads to an equivalent decomposition of the $a_{\mathbf{k},\lambda}(t)$ as well as of the fields $\mathbf{E}^{(+)}(\mathbf{r}, t)$ and $\mathbf{B}^{(+)}(\mathbf{r}, t)$. Thus the correlation function $w_{\mathcal{A},\mathbf{r}_0}(t_1, t_2)$ can be written as a sum $w_{\mathcal{A},\mathbf{r}_0} = w_{\mathcal{A},\mathbf{r}_0}^B + w_{\mathcal{A},\mathbf{r}_0}^I + w_{\mathcal{A},\mathbf{r}_0}^S$ analogous to Eq. (22) for w_α . In the following, we shall denote by $\langle \dots \rangle^{B,I,S}$ the sum of all the terms in $\langle \dots \rangle$ that contribute to $w_{\mathcal{A},\mathbf{r}_0}^{B,I,S}$, i.e., we set

$$\begin{aligned} \langle b_1^\dagger b_2 \rangle^B &= \langle b_1^{\dagger(1)} b_2^{(1)} \rangle & \langle b_1^\dagger b_2 \rangle^S &= \langle b_1^{\dagger(2)} b_2^{(2)} \rangle , \\ \langle b_1^\dagger b_2 \rangle^I &= \langle b_1^{\dagger(1)} b_2^{(2)} + b_1^{\dagger(2)} b_2^{(1)} \rangle , \end{aligned} \quad (52)$$

with b_i standing for $b_{\hat{\mathbf{k}}_i,\lambda_i}(t_i + s_i)_{t_i}$.

Using Eq. (50) the correlation function of the background part can easily be calculated. With

$$2 \Xi(\hat{\mathbf{k}}_1, \lambda_1; \hat{\mathbf{k}}_2, \lambda_2) \equiv \varepsilon_{\hat{\mathbf{k}}_1,\lambda_1} \times (\hat{\mathbf{k}}_2 \times \varepsilon_{\hat{\mathbf{k}}_2,\lambda_2}^*) - (\hat{\mathbf{k}}_1 \times \varepsilon_{\hat{\mathbf{k}}_1,\lambda_1}) \times \varepsilon_{\hat{\mathbf{k}}_2,\lambda_2}^* \quad (53)$$

one finds

$$\begin{aligned} w_{\mathcal{A},\mathbf{r}_0}^B(t_1 - t_2) &= \frac{\hbar\omega_0}{\lambda_0^2} \int_{(\mathcal{A},\mathbf{r}_0)} d\boldsymbol{\sigma} \cdot \int d^2\hat{\mathbf{k}}_1 \int d^2\hat{\mathbf{k}}_2 \sum_{\lambda_1, \lambda_2} \Xi(\hat{\mathbf{k}}_1, \lambda_1; \hat{\mathbf{k}}_2, \lambda_2) \\ &\quad \times \exp\left(-i\phi_{\hat{\mathbf{k}}_1,\lambda_1} + i\phi_{\hat{\mathbf{k}}_2,\lambda_2}\right) \exp\left(i\omega_0 c^{-1}(\hat{\mathbf{k}}_1 - \hat{\mathbf{k}}_2) \cdot \mathbf{r}\right) \\ &\quad \times \langle b_{\hat{\mathbf{k}}_1,\lambda_1}^\dagger(t_1 - c^{-1}\hat{\mathbf{k}}_1 \cdot \mathbf{r})_{t_1} b_{\hat{\mathbf{k}}_2,\lambda_2}(t_2 - c^{-1}\hat{\mathbf{k}}_2 \cdot \mathbf{r})_{t_2} \rangle^B \\ &= \delta(t_1 - t_2) \cdot \frac{\hbar\omega_0}{\lambda_0^2} \int_{(\mathcal{A},\mathbf{r}_0)} d\boldsymbol{\sigma} \cdot \int d^2\hat{\mathbf{k}} \sum_{\lambda} N_{\hat{\mathbf{k}},\lambda} \hat{\mathbf{k}} , \end{aligned} \quad (54)$$

where $\lambda_0 = 2\pi c/\omega_0$ is the wavelength corresponding to the atomic transition frequency.

The calculation of the interference and scattered part can be performed with the aid of the asymptotic expansion [18, 19]

$$\int d^2\hat{\mathbf{k}} e^{-i\mathbf{k} \cdot \mathbf{r}} f(\hat{\mathbf{k}}) \stackrel{r \rightarrow \infty}{\sim} \frac{2\pi i}{kr} \left\{ e^{-ikr} f(\hat{\mathbf{r}}) - e^{ikr} f(-\hat{\mathbf{r}}) \right\} + O(r^{-2}) \quad (55)$$

being valid as long as $f(\hat{\mathbf{k}})$ is sufficiently smooth. Note that this condition is violated for the background part [20] calculated above where $f(\hat{\mathbf{k}}) \propto \delta^2(\hat{\mathbf{k}}, \hat{\mathbf{k}}')$. By first applying (55) to either integral over $\hat{\mathbf{k}}$ in $w_{\mathcal{A}, \mathbf{r}_0}^{\text{I}, \text{S}}$, keeping only terms of the leading order r^{-1} , and then using Eq. (50), one obtains

$$\begin{aligned} w_{\mathcal{A}, \mathbf{r}_0}^{\text{I}, \text{S}}(t_1, t_2) &= \frac{\hbar\omega_0}{r^2} \int_{(\mathcal{A}, \mathbf{r}_0)} \mathbf{d}\boldsymbol{\sigma} \cdot \sum_{\xi_1=\pm 1} \sum_{\xi_2=\pm 1} \sum_{\lambda_1, \lambda_2} \xi_1 \xi_2 \Xi(\xi_1 \hat{\mathbf{r}}, \lambda_1; \xi_2 \hat{\mathbf{r}}, \lambda_2) \\ &\quad \times \exp(-i\phi_{\xi_1 \hat{\mathbf{r}}, \lambda_1} + i\phi_{\xi_2 \hat{\mathbf{r}}, \lambda_2}) \exp(i\omega_0 c^{-1}(\xi_1 - \xi_2)r) \\ &\quad \times \langle b_{\xi_1 \hat{\mathbf{r}}, \lambda_1}^\dagger(t_1 - c^{-1}\xi_1 r)_{t_1} b_{\xi_2 \hat{\mathbf{r}}, \lambda_2}(t_2 - c^{-1}\xi_2 r)_{t_2} \rangle^{\text{I}, \text{S}}. \end{aligned} \quad (56)$$

Due to

$$\Xi(\hat{\mathbf{k}}, \lambda_1; -\hat{\mathbf{k}}, \lambda_2) = 0 \quad \Xi(\hat{\mathbf{k}}, \lambda_1; \hat{\mathbf{k}}, \lambda_2) = \hat{\mathbf{k}} \delta_{\lambda_1 \lambda_2} \quad (57)$$

the sums are actually running only over $\xi_1 = \xi_2$ and $\lambda_1 = \lambda_2$. Further, since $\langle \dots \rangle^{\text{I}, \text{S}}$ contains at least one factor $\vartheta(t_i - t_i + c^{-1}\xi_i r) = \vartheta(\xi_i)$, only the term with $\xi_1 = \xi_2 = +1$ survives. For a surface which is sufficiently flat and whose diameter is small compared to its distance to the atom, \mathbf{r} can be approximated by \mathbf{r}_0 , the integration over $(\mathcal{A}, \mathbf{r}_0)$ resulting in a multiplication with the oriented area $\mathcal{A} = \int_{(\mathcal{A}, \mathbf{r}_0)} \mathbf{d}\boldsymbol{\sigma}$. So we finally have, with $w_{\hat{\mathbf{r}}, \lambda}^{\text{I}, \text{S}}$ defined as in Eqs. (29) and (30),

$$w_{\mathcal{A}, \mathbf{r}_0}^{\text{I}, \text{S}}(t_1, t_2) = \hbar\omega_0 \frac{\mathcal{A} \cdot \hat{\mathbf{r}}_0}{r_0^2} \sum_{\lambda} w_{\hat{\mathbf{r}}_0, \lambda}^{\text{I}, \text{S}}(t_1 - c^{-1}r_0, t_2 - c^{-1}r_0) \quad (58)$$

and

$$w_{\mathcal{A}, \mathbf{r}_0}^{\text{I}, \text{S}}(t) = \hbar\omega_0 \frac{\mathcal{A} \cdot \hat{\mathbf{r}}_0}{r_0^2} \sum_{\lambda} w_{\hat{\mathbf{r}}_0, \lambda}^{\text{I}, \text{S}}(t). \quad (59)$$

By inserting Eq. (41) one explicitly finds for γ , M , and N in Eq. (27)

$$\gamma = \frac{\omega_0^3 |\mathbf{d}|^2}{3\pi\epsilon_0 \hbar c^3}, \quad (60)$$

(this is the Einstein A coefficient for a dipole transition)

$$M = \frac{3}{8\pi} \int d^2\hat{\mathbf{k}} \sum_{\lambda} |\boldsymbol{\epsilon}_{\hat{\mathbf{k}}, \lambda} \cdot \hat{\mathbf{d}}|^2 M_{\hat{\mathbf{k}}, \lambda} \quad N = \frac{3}{8\pi} \int d^2\hat{\mathbf{k}} \sum_{\lambda} |\boldsymbol{\epsilon}_{\hat{\mathbf{k}}, \lambda} \cdot \hat{\mathbf{d}}|^2 N_{\hat{\mathbf{k}}, \lambda} \quad (61)$$

and the three parts of the spectrum read

$$\frac{2\pi}{\hbar\omega_0} \mathcal{S}_{\mathcal{A}, \mathbf{r}_0}^{\text{B}}(\omega) = \lambda_0^{-2} \int_{(\mathcal{A}, \mathbf{r}_0)} \mathbf{d}\boldsymbol{\sigma} \cdot \int d^2\hat{\mathbf{k}} \sum_{\lambda} N_{\hat{\mathbf{k}}, \lambda} \hat{\mathbf{k}} \quad (62)$$

$$\frac{2\pi}{\hbar\omega_0} \mathcal{S}_{\mathcal{A}, \mathbf{r}_0}^{\text{S}}(\omega) = \frac{3 \mathcal{A} \cdot \hat{\mathbf{r}}_0}{8\pi r_0^2} \frac{N\gamma}{2N+1} (1 - |\hat{\mathbf{d}} \cdot \hat{\mathbf{r}}_0|^2) \sum_{\pm} \frac{\gamma_{\pm}}{(\omega - \omega_0)^2 + \gamma_{\pm}^2} \quad (63)$$

$$\frac{2\pi}{\hbar\omega_0} \mathcal{S}_{\mathcal{A}, \mathbf{r}_0}^{\text{I}}(\omega) = \frac{3 \mathcal{A} \cdot \hat{\mathbf{r}}_0}{8\pi r_0^2} \sum_{\lambda} |\boldsymbol{\epsilon}_{\hat{\mathbf{r}}_0, \lambda} \cdot \hat{\mathbf{d}}|^2$$

$$\times \left\{ \frac{\gamma}{2N+1} \sum_{\pm} \frac{\gamma_{\pm}}{(\omega - \omega_0)^2 + \gamma_{\pm}^2} (-N_{\hat{\mathbf{r}}_0, \lambda} \mp \text{Re } M_{\hat{\mathbf{r}}_0, \lambda}) \right. \\ \left. + 2M\gamma^3 \frac{\omega - \omega_0}{[(\omega - \omega_0)^2 + \gamma_+^2][(\omega - \omega_0)^2 + \gamma_-^2]} \text{Im } M_{\hat{\mathbf{r}}_0, \lambda} \right\}, \quad (64)$$

with λ_0 the wavelength of the atomic transition frequency. We note the direction dependence of the spectrum and the $1/r^2$ dependence of $\mathcal{S}_{\mathcal{A}, \mathbf{r}_0}^{\text{S}}$ and $\mathcal{S}_{\mathcal{A}, \mathbf{r}_0}^{\text{I}}$.

Eq. (59) shows that, as far as the interference and scattered parts are concerned, the spectrum in Eqs. (62) – (64) is a sum of spectra for fixed polarization. The latter correspond to a spectrum for fixed α as in Section 2 and can be obtained by omitting the sum over λ in Eq. (64) and replacing the factor $1 - |\hat{\mathbf{d}} \cdot \hat{\mathbf{r}}_0|^2$ by $|\boldsymbol{\epsilon}_{\hat{\mathbf{r}}_0, \lambda} \cdot \hat{\mathbf{d}}|$ in Eq. (63).

The basic features of the spectra are the same as in the general case of Section 2. This will now be discussed in more detail.

4 Discussion

The spectra in Eq. (33), as well as those in Eqs. (62) – (64), are a sum of a background term \mathcal{S}^{B} , a scattered part \mathcal{S}^{S} and a term \mathcal{S}^{I} , which can be identified as an interference part. That this term is indeed due to interference (note that all radiation is quantized) can be seen in various ways. Formally, this is already suggested by Eqs. (21) and (22). The last term in Eq. (21) is due to the presence of the atom since it vanishes for $\gamma_{\alpha} = 0$ and Eq. (22) is due to the cross terms of this with b_{α} [21].

The interference nature of \mathcal{S}^{I} becomes yet more transparent if one uses the spectral Poynting vector for the calculation of the spectrum, as in Section 3. As pointed out there, the results for \mathcal{S}^{S} and \mathcal{S}^{I} remain the same for a direction-sensitive spectral analyzer, as long as the analyzer points in the direction of the atom. If the direction-sensitive analyzer does not point in this direction one shows by the same arguments as in Section 3 that \mathcal{S}^{S} and \mathcal{S}^{I} become zero. This is physically expected since it means that the corresponding light originates at the site of the atom. Furthermore, if the vacuum is squeezed only for directions in some cone \mathcal{C} , i.e., if $N_{\hat{\mathbf{k}}, \lambda}$ and $M_{\hat{\mathbf{k}}, \lambda}$ vanish for $\hat{\mathbf{k}}$ not in \mathcal{C} , then \mathcal{S}^{I} vanishes if the direction $\hat{\mathbf{r}}_0$ from atom to analyzer is not in the squeezing cone \mathcal{C} , since then $N_{\hat{\mathbf{r}}_0, \lambda}$ and $M_{\hat{\mathbf{r}}_0, \lambda}$ in Eq. (64) vanish. Moreover, \mathcal{S}^{I} depends on $N_{\hat{\mathbf{k}}, \lambda}$ and $M_{\hat{\mathbf{k}}, \lambda}$ only for $\hat{\mathbf{k}} = \hat{\mathbf{r}}_0$ [except for the general dependence of N and M in Eq. (61)], and this can be interpreted as the fact that scattered light interferes only with incident light traveling in the same direction, just as for classical light scattering. This can be traced back to Eq. (55).

The spectra in Eqs. (62) – (64) depend on the direction $\hat{\mathbf{r}}_0$ of the analyzer. As seen from Eq. (63), outside the squeezing cone the spectrum consists of a narrow Lorentzian sitting on top of a broad Lorentzian. This is a special case of the

dependence on α in Eq. (33). As remarked earlier there is no interference outside the squeezing cone. The analogous fact is true in the general case, as seen in Eq. (30) for \mathcal{S}_α^I which vanishes when the channel α is not squeezed.

For multi-channel squeezing one has more parameters available than in the single-channel case. In addition to N and M of the single-channel case one now also has N_α and M_α (or $N_{\mathbf{k},\lambda}$ and $M_{\mathbf{k},\lambda}$ in the plane-wave model), with $|M_\alpha|^2 = N_\alpha(N_\alpha + 1)$. The range of M and N is restricted by the inequality (28),

$$|M|^2 \leq N(N + 1), \quad (65)$$

where the equality sign holds, as easily shown, if and only if $N_\alpha \equiv N$ and $M_\alpha \equiv M$ for all α with $\gamma_\alpha \neq 0$. Only in this case does the spectrum $\mathcal{S}_\alpha(\omega)$ in Eq. (33) contain a peak which becomes increasingly narrow for increasing N as seen from the definition of γ_- in Eq. (31) which for constant ratio $m \equiv M[N(N + 1)]^{-1/2}$ can be written as

$$\gamma_- = \left[(N + \tfrac{1}{2})(1 - m) + \frac{m}{8N} + O(N^{-2}) \right] \gamma.$$

For $m = 1$ this not only recovers the one-dimensional like case of Gardiner [1] with its interesting narrow peak [22], but also shows that a subnatural linewidth and the above new features are mutually exclusive. A similar situation is found in Ref. [5] for single-channel squeezing with an additional laser.

If the equality does not hold in Eq. (65), then γ_- increases with increasing N and the corresponding peak cannot become arbitrarily narrow. But as long as $M^2/[N(N + 1)]$ is not too small (the spectra in figure 1 and 2 belong to $M^2/[N(N + 1)] = 0.75$), there are other interesting features in this case. First of all, if M_α is real then the spectrum is symmetric. However, if M_α is complex (since we have chosen M as positive this actually means M_α/M complex) then $\text{Im } M_\alpha \neq 0$ and the spectrum is asymmetric. But even if M_α is real, new phenomena occur, as seen in figure 1 and 2 for $\varphi = 0$. In this case the negative contribution can substantially exceed the positive one in absolute value, as shown in figure 2 ($\varphi = 0$), and for negative M_α/M the central maximum can be absent completely. The same is true in Eqs. (63) and (64) for the plane wave model.

If M_α/M is complex then the last term in Eq. (33), which comes from the interference part, makes the spectrum asymmetric. For the plane-wave model this is seen in Eq. (64). This asymmetry is a new phenomenon which does not occur in the one-dimensional case. Various typical spectra are shown in figure 1 and 2.

It seems that the relative phases of M and M_α play a similar role for the spectra as the introduction of an additional laser with a relative phase in the one-channel case, which has been investigated in Refs. [8, 9]. The spectra obtained in Ref. [9] resemble those in our figures. The similarity is particularly striking for figure 3 [23]. Spectra with a ‘pimple’ and central hole burning occur also

in Ref. [9], and we have a similar sensitivity of this effect on the parameters (in figure 3 the ‘pimple’ disappears for $\zeta = 0$, corresponding to the cancellation of the contributions from the scattered and interference part, respectively). It seems to us, however, that this is more a formal mathematical similarity of two physically distinct situations since in both cases one has sums and differences of Lorentzians and the possibility to adjust various parameters. Physically, the spectra calculated in Ref. [9] belong solely to the scattered light from the atom, which is driven by the combined field of the laser and the squeezed vacuum, and therefore these spectra correspond to detection directions away from the driving fields. In our case, however, these spectra result from the interference of the radiation emitted by the atom with the squeezed light traveling away from the atom, and in these directions the combined light is spectrally analyzed.

In summary, we have shown that in the case of an atom in a more general squeezed vacuum the spectrum can show new phenomena compared to the one-dimensional like case, among them absence of the positive peak and asymmetry. In the plane-wave model the shape of the spectrum can become direction dependent which, in the general case, is translated into α dependence. This dependence is due to an interference effect of the squeezed light with the scattered light.

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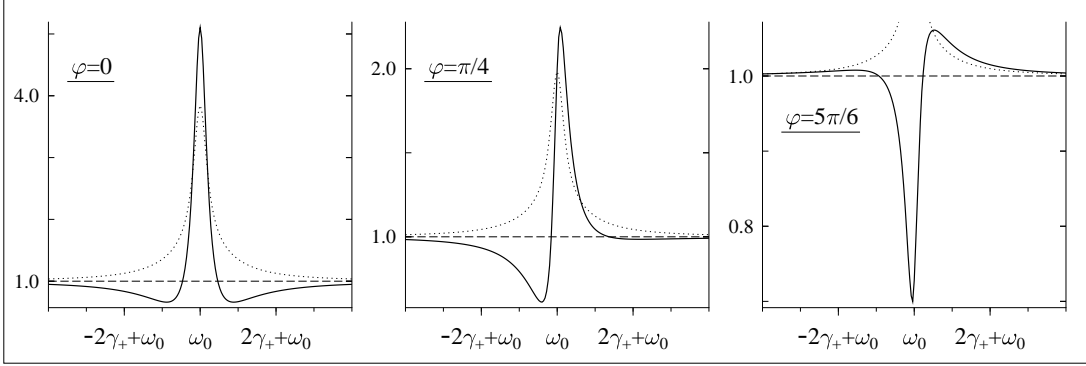


Figure 1: Typical spectra $\mathcal{S}_\alpha(\omega)$ for $N_\alpha = N = 0.25$, $M^2 = 0.75 N(N+1)$ and different phases φ of M_α (solid line). The dashed line denotes the background part of the spectrum, the dotted line the sum of the background and scattered part. The spectra belong to the largest values of γ_α/γ compatible with Eq. (27). The y axes are in units of $N_\alpha/2\pi$.

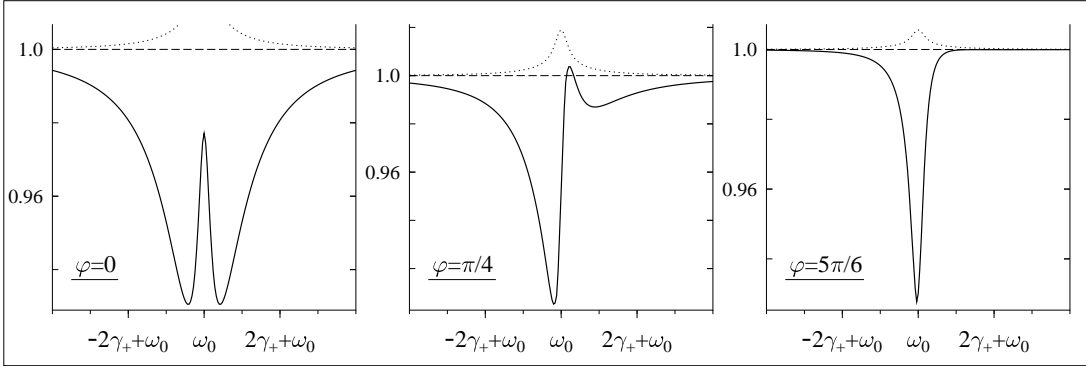


Figure 2: The spectra of figure 1, but with $N_\alpha = 8N$, $N = 0.25$, and $M^2 = 0.75 N(N+1)$.

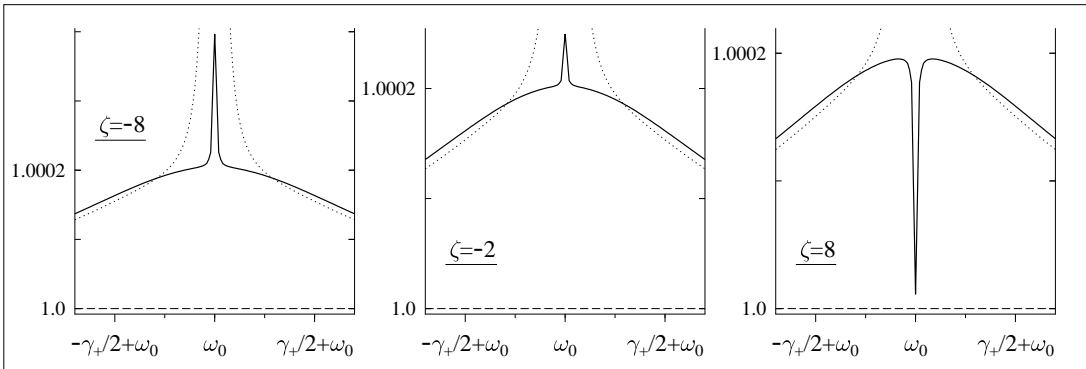


Figure 3: Spectra with $N_\alpha = (1 + \zeta/1000) \frac{N^2}{2N+1}$, $N = 5$, $M^2 = 0.98 N(N+1)$ and $\arg M_\alpha = \pi$.